Importance of distinction of levels in a logical discourse: an investigation from the perspective of a theory of graded consequence

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Abstract. In order to follow the objective of the title, at the outset let us put forward some quotations from the book 'Introduction to Mathematical Logic' by Alonzo Church. The quotations are scattered at different parts of the 'Introduction' of the book; we put them in a sequential order keeping the purpose of this abstract in mind. The quotations, mentioned below, are well enough to give a good account of the ideas we shall be venturing in. Our attempt in this abstract would be to bring to the fore the usual practice of the logical systems, where some of the following requirements are lacking. The theory of graded consequence, in contrast, would be presented as a formal set-up where the following prescriptions are preserved.

"*In order to set up a formalized language we must of course make use of a language already known to us, . . . The device of employing one language in order to talk about another is one for which we shall have frequent occasions not only in setting up formalized languages but also in making theoretical statements as to what can be done in a formalized language, our interest in formalized languages being less often in their actual and practical use as languages than in the general theory of such use and in its possibilities in principle. Whenever we employ a language to in order to talk about some language . . . we shall call the latter language the object language, and we shall call the former the meta-language.*"
[Church, p 47]

"*In defining a logistic system by laying down a primitive basis, we employ as meta-language the restricted portion of English . . . .*" [Church, p 50]

"*After setting up the logistic system as described, we still do not have a formalized language until an interpretation is provided. This will require a more extensive meta-language than the restricted portion of English used in setting up the logistic system. However, it will proceed not by translations of the well-formed formulas into English phrases but rather by semantical rules which, in general, use rather than mention English phrases . . . and which shall prescribe for every well-formed formula either how it denotes . . . or else how it has values . . . .*"
[Church, p 54]

"*The semantical rule must in the first instance be stated in a presupposed and therefore unformalized meta-language, here taken to be ordinary English. Subsequently, for their more exact study, we may formalize the meta-language (using a presupposed meta-meta-language and following the method already described for formalizing the objet language) and restate the semantical rules in this for-*

malized language (This leads to the subject of semantics) . . . . As a condition of rigor, we require that the proof of a theorem (of the object language) shall make no reference to or use of any interpretation, ..." [Church, p 55]
"*The study of the purely formal part of a formalized language in abstraction from the interpretation, i.e., of the logistic system, is called . . . logical syntax. The meta-language used in order to study the logistic system in this way is called the syntax language.*" [Church, p 58]
"*. . . the reader must always understand that syntactical discussions are carried out in a syntax language whose formalization is ultimately contemplated, and distinctions based upon such formalization may be relevant to the discussion. In such informal development of syntax, we shall think of the syntax language as being a different language from the object language.*" [Church, p 59]
"*Following the convenient and natural phraseology of Quine, we may distinguish between use and mention of a word or symbol. . . . As a precaution against univocation, we shall hereafter avoid the practice - which might otherwise sometimes be convenient - of borrowing formulas of the object language for use in the syntax language (or other meta-language) with the same meaning that they have in the object language.*" [Church, p 61-63]

The discussion would go in the following line.
Lecture - 1: Understanding the content and need of the above quotations in a logical discourse
Lecture - 2: Citing examples from usual practice in logical systems where some of the above needs are undermined, and presenting the difficulties which are arisen thereby
Lecture - 3: Presenting the Theory of graded consequence in the framework of distinction of levels of a logical discourse.

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